UNIT 5 *Probability* Activities

Activities

ACTIVITY 5.1 *Nearly the Nine O'Clock News*

"What might be on the news tonight? Write down four things which you think could be on the news tonight."

While pupils are writing down ideas, draw a probability line on board or OHP, labelled impossible, unlikely, even, likely, certain.

"Copy the line in your books, mark on the line where you think each of the items you have chosen will come on the line."

After pupils have written their ideas down, ask them to compare their line with a neighbour's.

"I'd like a volunteer to put one of their news items on the line on the board."

Invite individual pupils to put one of their ideas on the board. Encourage discussion of the placing of the event, using correct language. Which event is more likely? How much more likely? Draw out the need for more precision.

Redraw the line with ten divisions marked on it.

"Copy this line and put your news items on it."

Invite other pupils to put some of their items on the board against the scale.

Introduce the convention that impossible events have a probability of 0 and certain events have a probability of 1. Mark each point on the line as 0, 0.1, 0.2, ..., 0.9, 1.

Pupils then assign a value of 0, 0.1, 0.2, ..., 0.9, 1 to each of their news items.

There is a further opportunity for the work to be presented as a poster with a large probability line and pictures of current events in the news pasted on against the appropriate point on the line.

ACTIVITY 5.2 *Evens and Odds*

This is a simple game, where you throw a dice which controls the position of your counter on a 3×3 board.

FINISH		
		START

Place your counter at the START square. Throw a dice.

If you get an EVEN number, you move your counter one square *upwards*.

If you get an ODD number, you move your counter one square *left*.

If your counter moves off any side of the board, you lose!

If your counter reaches the FINISH square, you have won.

Play the game a few times and see if you win.

How many 'odds' and how many 'evens' do you need to get to win?

What is the probability of winning?

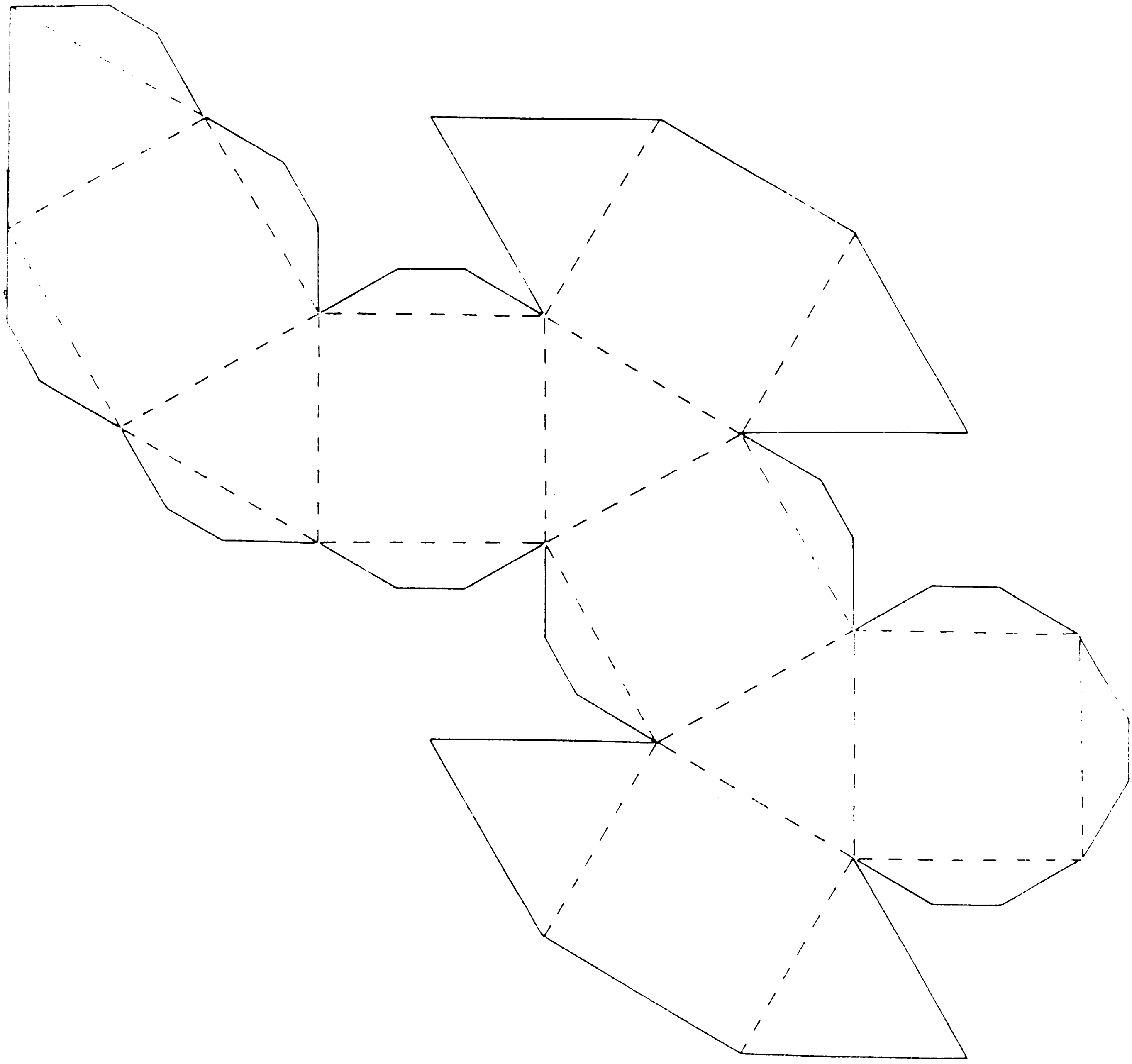
Extension

Analyse the same game on a 4×4, 5×5, . . ., board.

ACTIVITY 5.3 *Experimental Probability*

The net of a cuboctahedron is given below. It consists of 6 squares and 8 triangles.

Make this 3-dimensional object using card.



If this object is thrown, what do you think will be the probability of it landing on

(i) one of its square faces

(ii) one of its triangular faces?

Throw the object (at least 100 times) and estimate these probabilities.

How close are they to your original estimates?

ACTIVITY 5.4 *A Russian Fable*

This is the method traditionally used in some Russian villages to see which of the girls in the village are to be married next year! You take three blades of grass, folded in two, and hold them in your hand so that the six ends are hanging down. A young girl ties the ends together in pairs. If, on release, a large loop is formed, the girl will be married next year.

1. What are the possible outcomes for this experiment in terms of small, medium and large loops?

2. By labelling the six ends (say *a* and A for the two ends of one blade of grass), consider all the possible outcomes and hence find the probability of getting the large loop.

3. Test your predicted probabilities by using short lengths of string and getting the class to work in pairs, recording their answers. Collect all the data together and use it to work out the experimental probabilities. Compare these to the theoretical values found in question 2.

4. If a Russian village has 30 young girls and they all go through this ritual, how many do you estimate will be predicted to marry next year?

Extension

What happens if either 4 or 5 blades of grass (string) are used? What is the probability of now obtaining one large loop?

ACTIVITY 5.5 *Break the Bank*

Gamblers throughout history have tried to find systems to help them beat the bank. In most games of chance you will win eventually, but a long losing sequence can easily bankrupt you first.

Suppose we consider a very simple game – Heads and Tails – in which the bank will double your stake money if you win, but take your stake money if you lose.

Our objective is to win £10, and a possible sequence of events is shown below.

Stake	Win/Lose	Outcome	Balance	
£10	L	– £10	– £10	
£20	L	– £20	– £30	
£40	W	+£40	+£10	STOP, since we have now won £10

1. Draw up a table to show what happens when we have a sequence LLLLW. What is the maximum negative balance?

The difficulty with this system is that it might lead to a substantial deficit before you win. Of course, you will always win in the long run, but bankruptcy might come first! There are adaptations of this method which usually do not result in such large negative balances.

One method is to write £10 as £2 + 2 + 3 + 3 and follow the pattern shown below for the given results LLLLWWLWWLW.

STAKE is obtained from the sum of the two outside numbers of the sequence

If WIN, then delete the two outside numbers

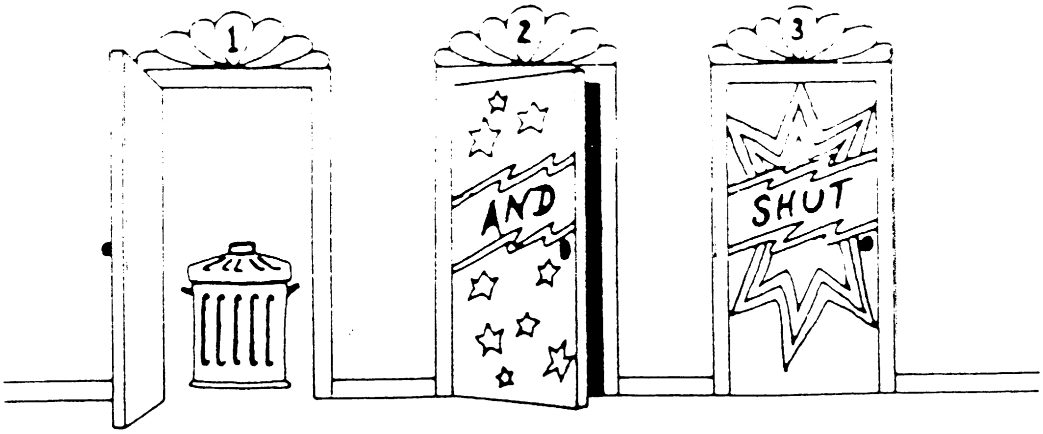
If LOSE, add in stake money to sequence

Sequence	Stake	Win/Lose	Outcome	Balance
2,2,3,3	5	L	–5	–5
2,2,3,3,5	7	L	–7	–12
2, 2,3,3,5,7	9	L	–9	–21
2,2,3,3,5,7,9	11	L	–11	–32
2,2,3,3,5,7,9,11	13	W	+13	–19
2,3,3,5,7,9	11	W	+11	–8
3,3,5,7	10	L	–10	–18
3,3,5,7,10	13	W	+13	–5
3,5,7	10	W	+10	+5
5,	5	L	–5	0
5, 5	10	W	+10	+10

So *eventually* you win and, despite the initial losing sequence, the overdraft is kept relatively small.

2. Play this game in pairs, trying different ways of writing £10 as the sum of four numbers. Does it make any difference writing £10 as a different sum?

ACTIVITY 5.6 *Open and Shut Case*



In a Game Show in America, the contestant is offered a choice of one of three doors to open. Behind one of these doors is the star prize, a *car,*– but behind the other two doors are *dustbins*!

Once the contestant has chosen say Door 2, the host, who already knows what is behind each door, opens one of the doors, say Door 1, to reveal a dustbin.

He then asks the contestant,

"Do you want to stick with your original choice (Door 2) or switch to the other closed door (Door 3)?"

1. Is it to your advantage to change your choice from Door 2 to Door 3?

It is easy to provide an argument for either policy.

ARGUMENT 1 When one door is opened, there is an equal chance of the car being behind either of the other two doors, so there is no need to change.

ARGUMENT 2 There is a *2* in *3* chance of being *wrong* initially. If you were wrong and changed, you would now be right, so the probability is reversed and you will now be *right* 2 out of 3 times.

2. Simulate this Game Show by playing it with a partner. One of you is the contestant and the other the Game Show host. You will need to play the game at least 20 times in order to gain insight into the solution

If this simulation does not convince you, then try using a *computer program* to simulate the situation 10 000 or 20 000 times.

Extension

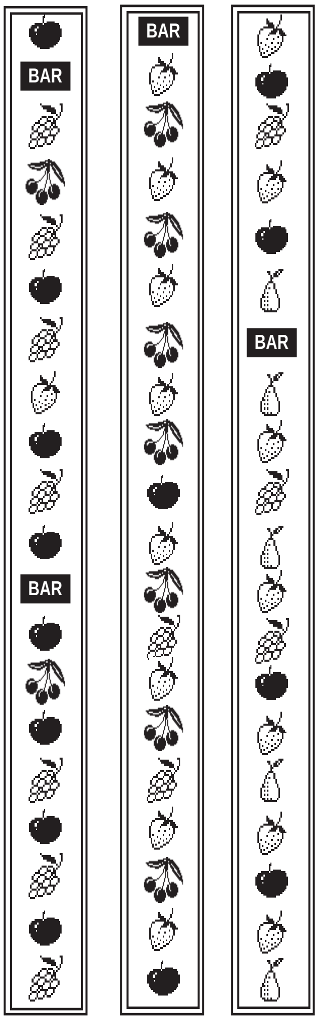
Suppose there are now *four* doors with a star prize behind one door and dustbins behind each of the other doors.

Again the contestants are offered the chance of changing their choices.

Should they change, and if they do, what is now their probability of winning?

ACTIVITY 5.7 *Fruit Machines*

A fruit machine with 3 DIALS and 20 SYMBOLS (not all different) on each dial is illustrated opposite. Each dial can stop on any one of its 20 symbols, and each of the 20 symbols on a dial is equally likely to occur.



So, for example, the *Grapes* on DIAL 1 are likely to occur on average 7 times out of 20.

You inset 10p, press a button and the three dials spin round. You then press more buttons to stop each dial at random.

The three symbols highlighted determine how much, if anything, is won.

For example, Suppose the machine makes the payouts shown opposite.

Combination	**Payout** (in 10p's)
3 BARS	40
3 STRAWBERRIES	5
3 GRAPES	5
3 APPLES	5
2 BARS	20
2 CHERRIES	5

1. Copy and complete the frequency chart below for each dial.

Symbol	**Dial 1**	**Dial 2**	**Dial 3**
BAR	2	1	1
STRAWBERRY	1	8	..
GRAPE	7	..	..
APPLE	..	..	..
CHERRY	..	..	..
PEAR	..	..	..

We want to find the probability of each of the combinations above to see if it is worth playing. We first consider the 3 BARS combination.

2. (a) In how many ways can you obtain 3 BARS?
   (b) How many possible combinations (including repeats) are there?
   (c) What is the probability of obtaining 3 BARS?

We can find the probability of the other winning combinations in the same way.

3. Find the probabilities of obtaining all the other winning combinations.

Your expected winnings in 10 pences are

$$40 \times (\text{probability of 3 BARS}) + 5 \times (\text{probability of 3 STRAWBERRIES}) + \ldots\ldots$$

but you must take off your initial payment of 10 pence.

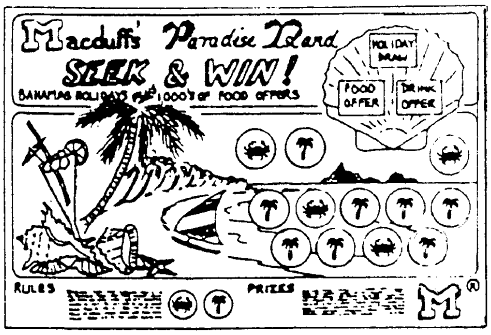
4. What is the expected gain or loss for each go?

Extension

Design your own fruit machine, work out the probabilities of certain combinations, assign payouts and check whether the player expects to gain or lose money.

ACTIVITY 5.8 *Seek and Win*

The fast food chain, *Macduff's*, is running a competition. You obtain a card which has 12 circles covered up and you can scratch off up to four circles.
You win if 3 or more PALM TREES are revealed but lose if 2 or more CRABS are revealed. If you win you can then scratch off one of the three squares to show what you have won.



One of the cards is shown above with *all* the circles and *all* the squares revealed. *Macduff's* want the game to be both fun to play and relatively easy to win. Here we find out your chance of winning if the PALM TREES and CRABS are always in the ratio 2 : 1.

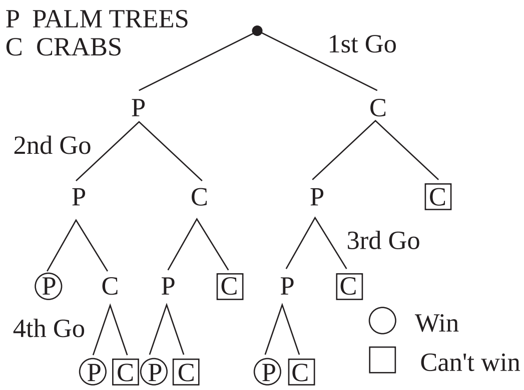
1. Check that the ratio of PALM TREES to CRABS is 2 : 1 on the picture above.

2. If all the circles are now covered up, on your first choice what is the probability of revealing
   (a) a PALM TREE (b) a CRAB?

If you obtained a PALM TREE on your first go, there are now 11 circles to be revealed, of which 7 are PALM TREES and 4 are CRABS.

3. What is the probability of revealing (a) a PALM TREE (b) a CRAB
   on your second go if your revealed a PALM TREE on your first go?

We can continue in this way –
a *tree diagram* is very helpful.



4. On each of the lines opposite, write down the probability of each event. For example,

$$p(\text{PCPP}) = \frac{8}{12} \times \frac{4}{11} \times \frac{7}{10} \times \frac{6}{9}.$$

5. Find the probability of each winning combination. (There are 4 possibilities.)

6. Find the probability of winning.

Extension
Work through the problems again, assuming there are now
(a) 9 PALM TREES and 3 CRABS (b) 6 PALM TREES and 6 CRABS.

ACTIVITY 5.9(a) *Misconceptions*

Misconceptions about probability may include:

- All events are equally likely
- Later events may be affected by or compensate for earlier ones
- When determining probability from statistical data, sample size is irrelevant
- Results of games of skill are unaffected by the nature of the participants
- 'Lucky/Unlucky' numbers, etc. can influence random events
- In random events involving selection, results are dependent on numbers rather than ratios
- If events are random then the results of a series of independent events are equally likely, e.g. HH is as likely as HT
- When considering spinners, probability is determined by number of sections rather than size of angles.

This activity is intended to provide an opportunity to discuss common misconceptions. The statements given are all incorrect. They can be copied onto card, cut into individual statements and given to pairs of pupils to discuss. Each pair can then explain their statement and the error to the whole group.

Alternatively, pupils can be given the complete set of statements and, after they have had time to consider them, the statements can be discussed by the class, or they can be used on an OHP for whole class discussion.

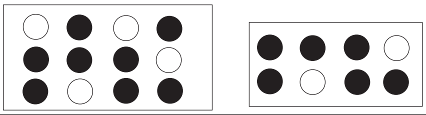
ACTIVITY 5.9(b) *Misconceptions*

1. I've spun an *unbiased* coin 3 times and got 3 heads. It is more likely to be tails than heads if I spin it again.	**2.** Aytown Rovers play Betown United. Aytown can win, lose or draw, so the probability that Aytown will win is $\frac{1}{3}$.
3. There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$.	**4.** I roll two dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.
5. It is harder to throw a six than a three with a die.	**6.** Tomorrow it will either rain or not rain, so the probability that it will rain is 0.5.
7. Mr Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr Brown will make a complete recovery if he has this operation.	**8.** If six fair dice are thrown at the same time, I am less likely to get 1, 1, 1, 1, 1, 1 than 1, 2, 3, 4, 5, 6.

ACTIVITY 5.9(b) *Misconceptions*

9.

There are more black balls in box A than in box B. If you chooses 1 ball from each box you are more likely to choose a black ball from A than from B.

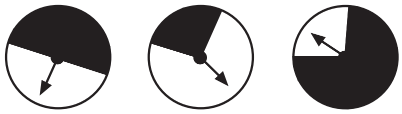
A  B

10.

I spin two coins. The probability of getting heads and tails is $\frac{1}{3}$ because I can get Heads and Heads, Heads and Tails or Heads and Tails.

11.

John buys 2 raffle tickets. If he chooses two tickets from different places in the book he is more likely to win than if he chooses two consecutive tickets.

12. 

Each spinner has two sections – one black and one white. The probability of getting black is 50% for each spinner.

13.

13 is an unlucky number so you are less likely to win a raffle with ticket number 13 than with a different number.

14.

My Grandad smoked 20 cigarettes a day for 60 years and lived to be 90, so smoking can't be bad for you.

15.

It is not worth buying a national lottery card with numbers 1, 2, 3, 4, 5, 6, on it as this is less likely to occur than other combinations.

16.

I have thrown an unbiased dice 12 times and not yet got a six. The probability of getting a 6 on my next throw is more than $\frac{1}{6}$.

ACTIVITY 5.10 *Birthdays*

First try this experiment. Find out the birthdays of as many of your family as possible. Do any of them have birthdays on the same day of the year?

Now try the same experiment with all the members of your class. We will see how likely it is that two members of a group have the same birthday.

Consider each member of a group, one by one. The first person will have his/her birthday on a particular day.

1. What is the probability of the second person having a different birthday from the first?

2. What is the probability of the third person having a birthday different from both the first and second person?

3. What is the probability that at least two of the first three people have the same birthday?

This solves the problem of a group of three people. As expected, it is not likely that any 2 out of 3 people will have the same birthday.

4. Repeat the problem above for 4 people. What is the probability that at least 2 of them have the same birthday?

5. Using either a computer programme or a calculator, solve the problem for a group of n people, where $n = 10, 20, 30$, etc.

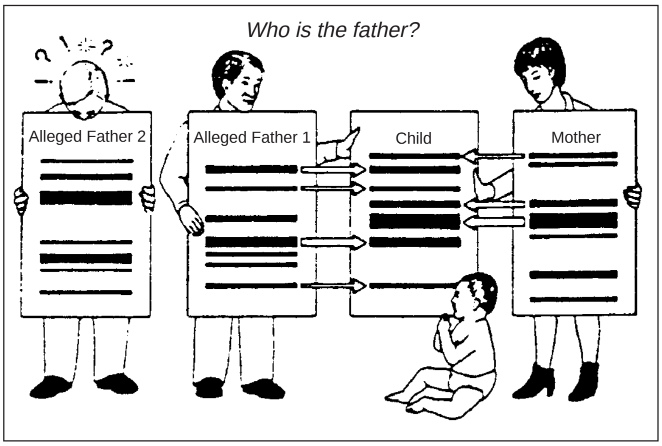
6. What is the probability that 2 members of your class have the same birthday?

Extension

How many people are needed in the group to be 95% sure that there will be at least two with the same birthday?

ACTIVITY 5.11 *Genetic fingerprinting*

Genetic fingerprinting was developed by Professor Alec Jeffreys at the University of Leicester in 1984. The technique is based on the fact that each of us has a unique genetic make up, contained in the molecule DNA, which is inherited from our natural parents, half from our mother and half from our father.
DNA can be extracted from cells to and body fluids and analysed to produce a characteristic pattern of bands or genetic 'fingerprint. The sketch below shows how our genetic fingerprinting can be used to identify a child's father.



Equally important has been the use of genetic fingerprinting in rape cases, where the semen of the attacker and the alleged rapist can be compared.

It is usual to compare between 10 and 20 bands. Experimental evidence has shown that in unrelated people the probability of one band matching is one in four (0.25); so for example, the probability of two bands matching $= (0.25)^2 = 0.0625$, a 1 in 16 chance.

1. Find the probability of 10 bands matching. Express your answer in the form "1 in ? chance".

2. Repeat the above, but using 0.5 as the probability of any single band matching.

You will have noticed that the answers to problem 1 and 2 change quite dramatically if the underlying probability changes. In fact, the value of 0.25 has been the subject of some speculation recently in a number of criminal trials.

3. Copy and complete the table below. Comment on the values found and suggest the number of bands which should be compared, to be confident of a match not happening by chance, when the probability is 0.25.

Probability (p)	**Number of bands compared** 5	10	15	20
0.2	1 in 3125	?	?	1 in 9.5 million million
0.25	?	?	?	?
0.5	?	?	?	?

Extension

If p = 0.25 and we wish the probability of a complete match not happening by chance to be 1 in 50 million (approximately the population of Britain), how many bands need to be compared?